

# Mean Field Dynamo Saturation: Toward Understanding Conflicting Results

Eric G. Blackman

*Department of Physics & Astronomy, University of Rochester,  
Rochester, NY 14627*

George B. Field

*Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138*

**Abstract.** Mean field dynamos may explain the origin of large scale magnetic fields of galaxies, but controversy arises over the extent of dynamo quenching by the growing field. Here we explain how apparently conflicting results may be mutually consistent, by showing the role of magnetic helicity conservation and boundary terms usually neglected. We estimate the associated magnetic energy flowing out of the Galaxy but emphasize that the mechanism of field escape needs to be addressed.

## 1. Field Growth and Constraining the Turbulent EMF

Unlike the turbulent amplification of small scale magnetic energy to near equipartition with the the kinetic energy spectrum, the mean field dynamo (MFD) field generation (c.f. Parker 1979, Kulsrud 1999) on scales  $>$  turbulent input scale is controversial (c.f. Field et al. 1999). The MFD equation is  $\partial_t \bar{\mathbf{B}} = \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle + \lambda \nabla^2 \bar{\mathbf{B}} + \nabla \times (\bar{\mathbf{V}} \times \bar{\mathbf{B}})$ , with the turbulent EMF  $\langle \mathbf{v} \times \mathbf{b} \rangle = \alpha \bar{\mathbf{B}} - \beta \nabla \times \bar{\mathbf{B}}$ , and pseudo-scalar  $\alpha$  and scalar  $\beta$ . How well MFD growth applies when the when the dynamic magnetic backreaction is included depends on the survival of  $\langle \mathbf{v} \times \mathbf{b} \rangle$ . Blackman & Field (2000a) used Ohm's law and mean field theory (e.g.  $\mathbf{B} = \mathbf{b} + \bar{\mathbf{B}}$ ;  $\langle \mathbf{b} \rangle = 0$ ) to constrain the dynamic value of  $\langle \mathbf{v} \times \mathbf{b} \rangle$  analytically. Deriving  $\langle \mathbf{v} \times \mathbf{b} \rangle \cdot \bar{\mathbf{B}} / c = -\eta \langle \mathbf{j} \cdot \mathbf{b} \rangle + \langle \mathbf{e} \cdot \mathbf{b} \rangle$  and then expanding the fluctuating electric field  $\mathbf{e}$  into its potentials, gives  $\langle \mathbf{e} \cdot \mathbf{b} \rangle = -\partial_t \langle \mathbf{a} \cdot \mathbf{b} \rangle / 2c + \nabla \cdot \langle \mathbf{a} \times \mathbf{e} - \phi \mathbf{b} \rangle$ . Thus, for  $\langle \mathbf{v} \times \mathbf{b} \rangle$  not to be resistively limited, there must be time variation of  $\langle \mathbf{a} \cdot \mathbf{b} \rangle$ , or non-vanishing boundary terms. When such terms vanish, helical turbulence without mean field gradients gives  $\alpha \leq (b/\bar{B})^2 \alpha_0 / R_m$ , where  $\alpha_0$  is the kinematic value of the pseudoscalar coefficient  $\alpha$ , and  $R_m$  is the magnetic Reynolds number. There is thus an ambiguity in interpreting all existing numerical experiments suggesting  $\alpha$  quenching (e.g. Cattaneo & Hughes 1996); the quenching might not be dynamical, but may be due to boundary conditions.

## 2. Magnetic Helicity Escape, Dynamo Action, & Coronal Activity

The above result highlights the role of total magnetic helicity  $H^M = \int_V \mathbf{A} \cdot \mathbf{B} d^3x$  (Elsässer 1956), where  $V$  is a volume of integration, and  $\mathbf{A}$  is the vector potential.

That MFD growth involves a magnetic helicity inverse cascade was demonstrated by Pouquet *et al.* (1976). The  $\alpha$  effect conserves  $H^M$  by pumping a positive (negative) amount to scales  $> L$  (the outer turbulent scale) and a negative (positive) amount to scales  $\ll L$ . Brandenburg's (2000) simulations confirm this inverse cascade and the role of  $H^M$  conservation.

A large-scale field can be generated only as fast as  $H_M$  can be removed or dissipated. Presently, simulations have invoked boundary conditions for which the growth of large scale field is resistively limited. Large  $R_m$  systems must rely on open boundary conditions. To see this, note that  $H^M$  satisfies  $\partial_t(\mathbf{A} \cdot \mathbf{B}) + c\nabla \cdot (\mathbf{E} \times \mathbf{A} + A_0\mathbf{B}) = -2c\mathbf{E} \cdot \mathbf{B}$  where  $\mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B}$ . Consider two cases. Case (1): The mean scale = universal scale, or the integration is over periodic boundaries. Then boundary terms vanish, so  $\partial_t\langle\mathbf{A} \cdot \mathbf{B}\rangle = -2c\langle\mathbf{E} \cdot \mathbf{B}\rangle = -2c\overline{\mathbf{E}} \cdot \overline{\mathbf{B}} - 2c\langle\mathbf{e} \cdot \mathbf{b}\rangle = \eta\langle\mathbf{J} \cdot \mathbf{B}\rangle$  and  $\partial_t\langle\overline{\mathbf{A}} \cdot \overline{\mathbf{B}}\rangle = -2c\overline{\mathbf{E}} \cdot \overline{\mathbf{B}}$ ;  $\partial_t\langle\mathbf{a} \cdot \mathbf{b}\rangle = -2c\langle\mathbf{e} \cdot \mathbf{b}\rangle$ . This is the case of section 1. Dynamo action is resistively limited. Case (2): The system (e.g. Galaxy or Sun) mean volume  $V \ll$  universal volume. Here we must use the gauge invariant relative helicity  $H_R^M$  inside and outside of the spherical or disk rotator (Berger & Field 1984). The integral over the universal volume then satisfies  $\partial_t \int_U \mathbf{A} \cdot \mathbf{B} d^3x = \partial_t H_{R,in}^M + \partial_t H_{R,out}^M = -2c \int_U \mathbf{E} \cdot \mathbf{B} d^3x \simeq 0$ . The formulae for the  $H_R^M$  of the mean and fluctuating quantities inside the rotator are  $\partial_t H_{R,in}(\overline{\mathbf{B}}) = -2c \int_{in} \overline{\mathbf{E}} \cdot \overline{\mathbf{B}} d^3x + 2c \int_{S_{in}} (\overline{\mathbf{A}}_p \times \overline{\mathbf{E}}) \cdot d\mathbf{S}$  and  $\partial_t H_{R,in}(\mathbf{b}) = -2c \int_{in} \langle\mathbf{e} \cdot \mathbf{b}\rangle d^3x + 2c \int_{S_{in}} \langle\mathbf{a}_p \times \mathbf{e}\rangle \cdot d\mathbf{S}$ . In a steady state,  $\partial_t H_{R,in} = 0 = \partial_t H_{R,out}$ . This and  $\mathbf{E} \cdot \mathbf{B} \simeq 0$  imply that the above surface terms above must be equal and opposite. Moreover, the surface term balances the  $\overline{\mathbf{E}} \cdot \overline{\mathbf{B}}$  term in the  $\partial_t H_{R,in}^M$  equation. The boundary term can thus allow for a significant turbulent EMF because the latter is contained in  $\overline{\mathbf{E}} \cdot \overline{\mathbf{B}}$ . Dynamo action unrestricted by resistivity is possible only in case (2). This is consistent with Pouquet *et al.* (1976) and Brandenburg (2000).

If  $H_M$  flows through the boundary, then so does magnetic energy. Blackman & Field (2000b) showed that a typical minimum power leaving the system when a MFD is operating is given by  $\dot{E}^M \geq \frac{k_{\min}}{8\pi} |\dot{H}^M| = \frac{k_{\min}}{6} |\langle\alpha\bar{B}^2\rangle| V$ , where  $V$  is the system volume. Dynamos operating in the Sun, accretion disks, and the Galaxy would then lead to a net escape of magnetic energy and small and large scale magnetic helicity. Coronal activity from the emergence and dissipation of helical magnetic flux is thus a prediction of the MFD in all of these cases, and is observed directly in the Sun (c.f. Pevtsov *et al.* 1999). For the Galaxy,  $\dot{E}^M \gtrsim (\pi R^2)\alpha\bar{B}^2 \sim 10^{40}(R/12\text{kpc})^2(\alpha/10^5\text{cm/s})(\bar{B}/5 \times 10^{-6}\text{G})^2\text{erg/s}$ , in each hemisphere. Blackman & Field (2000b) discuss how this relation may be consistent with coronal energy input rates required by Savage (1995) and Reynolds *et al.* (1999).

### 3. Open Questions

An MFD unlimited by resistivity requires the helicity to flow through the boundary AND that there be some mechanism that enables this flow. Thus there are two separate issues. Even if the boundary conditions allow it, does it actually happen? One may have to include the dynamics of buoyancy or winds to fully demonstrate the non-resistive MFD. Note that turbulent diffusion of

the mean magnetic field (not necessarily the actual field) across the boundary is required to maintain a quadrupole field in the Galaxy with a net flux inside the disk. Similarly, for the Sun, the solar cycle requires net diffusion through the boundary. The flow of helicity would appeal to the same dynamics needed by these constraints.

The analytic and numerical studies that we have seen which show catastrophic suppression of the dynamo coefficients, or resistively limited dynamo action, either (1) invoke periodic boundary conditions, and/or (2) are 2-D, or (3) do not distinguish between zeroth order isotropic components of the turbulence and the higher order anisotropic perturbations for a weak mean field (Blackman & Field 1999). This means that there always seems to be an alternative explanation. and the observed suppression is then ambiguous as we have described. This does not mean that some of the physical concepts found in the strong suppression results are invalid, but just that they may be valid only for the restricted cases considered. For example, the observation that the Lagrangian chaos properties of the flow are changed in the presence of a weak mean field for turbulence in a periodic box (e.g. Cattaneo et al. 1996) needs to be understood in the relation to the imposed boundary conditions, and the shape of the magnetic energy spectrum (e.g. dominated at small or large scales?). Along these lines, note that the helicity constraint is global, but also becomes a constraint for any sub-volume of a periodic box once the system is fully mixed.

#### 4. References

- Blackman, E. G. & Field, G. B. 1999, ApJ **521** 597.  
 Blackman, E. G. & Field, G. B. 2000a, ApJ **534** 984.  
 Blackman, E. G. & Field, G. B. 2000b, MNRAS, in press, astro-ph/9912459.  
 Berger M.C. & Field G.B., 1984, JFM, **147** 133.  
 Brandenburg A., 2000, submitted to ApJ, astro-ph/0006186.  
 Cattaneo, F., & Hughes, D.W. 1996, Phys. Rev. E. **54**, 4532.  
 Cattaneo, F., & Hughes, D.W., Kim E.-J, 1996, PRL **76**, 2057.  
 Elsässer, W. M. 1956, Rev. Mod. Phys. **23**, 135.  
 Field, G. 1986, in *Magnetospheric Phenomena in Astrophysics*, R. Epstein & W. Feldman, eds., AIP Conf. Proc. 144 (Los Alamos: Los Alamos Sci. Lab.), 324.  
 Field, G. B., Blackman, E. G., & Chou, H. 1999, ApJ **513**, 638.  
 Frisch, U., Pouquet, A., L  orat, J. & Mazure, A. 1975, JFM **68**, 769.  
 Krause F. & R  dler K.-H., 1980, *Mean-field magnetohydrodynamics and dynamo theory*, (New York: Pergamon).  
 Kulsrud R., 1999, ARAA, **37** 37.  
 Parker, E. N., *Cosmical Magnetic Fields* (Oxford: Clarendon Press).  
 Pevtsov, A., Canfield, R. C., & Brown, M. R. eds. 1999, *Magnetic Helicity in Space and Laboratory Plasmas*, (American Geophysical Union).  
 Pouquet, A., Frisch, U., & Leorat, J. 1976, JFM **77**, 321.  
 Reynolds R.J., Haffner L.M., Tufte S.L., 1999, ApJ **525** 21.  
 Savage B.D., 1995, in *The Physics of the Interstellar Medium and Intergalactic Medium*, A. Ferrara, C.F. McKee, C. Heiles, & P.R. Shapiro eds. ASP conf ser vol 60. (San Francisco: PASP) p233.